54-19-75

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3185

TABLES FOR THE COMPUTATION OF WAVE DRAG OF ARROW WINGS OF ARBITRARY AIRFOIL SECTION

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Washington
June 1954

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SUMMARY

Tables and computing instructions are presented for the rapid evaluation of the wave drag of delta wings and of arrow wings having a ratio of the tangent of the trailing-edge sweep angle to the tangent of the leading-edge sweep angle in the range from -1.0 to 0.8. The tables cover a range of both subsonic and supersonic leading edges.

INTRODUCTION

The computation of the wave drag of wings at supersonic speeds is an extremely tedious task when other than the simplest of airfoil sections are considered. In reference 1, the basic equations for the drag of arrow wings of double-wedge airfoil sections have been obtained by the superposition of constant-strength source distributions within the wing plan form. This method has been extended in reference 2 into a generalized procedure whereby the wave drag of arrow wings having arbitrary profiles may be determined. This extension is accomplished by using a finite number of constant-strength source distributions and hence entails approximating the airfoil section by a multisided polygon. This approach was used (ref. 3) to determine minimum-drag airfoil sections for wings of a given volume or a given thickness ratio at a specified chordwise position, subject to the additional restriction that the airfoil section be the same at all spanwise stations. For the configurations studied in reference 3, computations were required for generalized coefficients which can be used for the rapid evaluation of the wave drag of arrow wings having airfoil sections represented by twenty equally spaced straight-line segments per surface. It is the purpose of this paper to present these coefficients, together with additional computations covering a wide range of the ratio of the tangent of the trailing-edge sweep angle to the tangent of the leading-edge sweep angle for both subsonic and supersonic leading edges. The use of these coefficients is explained and an illustrative example presented.

The authors wish to acknowledge the cooperative efforts of the National Bureau of Standards personnel, particularly Otto Steiner, Sally Tsingou, and Norman Levine, who set up the coding and carried through the numerical computations on the National Bureau of Standards Eastern Automatic Computer during June 1953.

SYMBOLS

M	Mach number
β	cotangent of Mach angle, $\sqrt{M^2 - 1}$
k_1	tangent of leading-edge sweep angle (see fig. 1)
k_{N+1}	tangent of trailing-edge sweep angle (see fig. 1)
$n_1 = k_1/\beta$	
N	number of equally spaced straight-line segments used to form one side of symmetrical airfoil
C_D	drag coefficient based on wing area
t	thickness ratio
x,y,z	Cartesian coordinates
λ_i	slope of airfoil surface divided by thickness ratio, $\dfrac{z_{i+1} - z_i}{t\left(x_{i+1} - x_i\right)}$
i,j	arbitrary indices
$\Delta\lambda_i = \lambda_i - \lambda_{i-1}$	
m	integer specifying location of maximum thickness

BASIS OF TABLES

The wave drag of an arrow wing given by linear supersonic theory in reference 3 can be written in the form

$$-\frac{\beta C_D}{t^2} = \sum_{i=1}^{N} \sum_{j=i+1}^{N+1} C_{ij}\, \Delta\lambda_j\, \Delta\lambda_i \tag{1}$$

where C_{ij} is a function of

N number of equally spaced straight-line segments per surface used to approximate the airfoil

k_{N+1}/k_1 ratio of the tangent of the trailing-edge sweep angle to the tangent of the leading-edge sweep angle

n_1 ratio of the tangent of the leading-edge sweep angle to the tangent of the Mach line sweep angle

The specific formula for C_{ij} can be determined from reference 3.

Values of C_{ij} for $N = 20$ are presented in tables I to VIII for a range of values of k_{21}/k_1 and n_1. (These tables are presented in loose form in the envelope at the end of this report.) A complete summary of the specific values assumed and the table in which the values of C_{ij} are given is presented in the following table:

k_{21}/k_1	Table									
	$n_1 = 0.25$	$n_1 = 0.50$	$n_1 = 0.75$	$n_1 = 1.00$	$n_1 = 1.25$	$n_1 = 1.30$	$n_1 = 10/6$	$n_1 = 1.70$	$n_1 = 2.50$	$n_1 = 5.00$
-1.0	I(a)	I(b)	I(c)	I(d)						
-.6	II(a)	II(b)	II(c)	II(d)		II(e)	II(f)			
-.4	III(a)	III(b)	III(c)	III(d)		III(e)		III(f)	III(g)	
-.2	IV(a)	IV(b)	IV(c)	IV(d)		IV(e)		IV(f)	IV(g)	IV(h)
0	V(a)	V(b)	V(c)	V(d)		V(e)		V(f)	V(g)	V(h)
.2	VI(a)	VI(b)	VI(c)	VI(d)		VI(e)		VI(f)	VI(g)	VI(h)
.4	VII(a)	VII(b)	VII(c)	VII(d)		VII(e)		VII(f)	VII(g)	
.8	VIII(a)	VIII(b)	VIII(c)	VIII(d)	VIII(e)			VIII(f)		

For the sweptforward trailing edges (tables I to IV), the largest value of n_1 is that which gives sonic trailing edges. For compactness of presentation, coefficients for two values of n_1 are presented per page, one being upside down. The values are given to seven decimal places, except in table I(d) in the first row and the last column where the calculations are accurate to only five decimal places.

USE OF TABLES

The use of the tabulated coefficients for the evaluation of $\beta C_D/t^2$ is illustrated for a five-sided airfoil ($N = 5$) by the following simplified table which is set up to facilitate computations on a desk-type computing machine equipped with an accumulative-multiplication feature:

Σ_1	Σ_2	Σ_3	Σ_4	Σ_5	
$\Delta\lambda_1$	$\Delta\lambda_2$	$\Delta\lambda_3$	$\Delta\lambda_4$	$\Delta\lambda_5$	$\Delta\lambda_6$
	A	B	C	D	E
		F	G	H	I
			J	K	L
				N	P
					Q

The coefficients A to Q are the tabulated C_{ij} values. The $\Delta\lambda$'s are airfoil-geometry parameters computed on an auxiliary form from the relationship:

$$\Delta\lambda_i = \lambda_i - \lambda_{i-1}$$

where

$$\lambda_i = \frac{z_{i+1} - z_i}{t\left(x_{i+1} - x_i\right)}$$

The parameter $\sum_i$ is

$$\sum\nolimits_1 = A\Delta\lambda_2 + B\Delta\lambda_3 + C\Delta\lambda_4 + D\Delta\lambda_5 + E\Delta\lambda_6$$

$$\sum\nolimits_2 = F\Delta\lambda_3 + G\Delta\lambda_4 + H\Delta\lambda_5 + I\Delta\lambda_6$$

$$\sum\nolimits_3 = J\Delta\lambda_4 + K\Delta\lambda_5 + L\Delta\lambda_6$$

$$\sum\nolimits_4 = N\Delta\lambda_5 + P\Delta\lambda_6$$

$$\sum\nolimits_5 = Q\Delta\lambda_6$$

In performing the calculations with desk-type machines with the accumulative-multiplication feature, only the final $\sum_i$ values need be recorded. The drag is then given by (eq. (1)):

$$-\frac{\beta C_D}{t^2} = \Delta\lambda_1\sum\nolimits_1 + \Delta\lambda_2\sum\nolimits_2 + \Delta\lambda_3\sum\nolimits_3 + \Delta\lambda_4\sum\nolimits_4 + \Delta\lambda_5\sum\nolimits_5 \qquad (2)$$

ILLUSTRATIVE EXAMPLE

A detailed calculation is now performed for illustrative purposes for $n_1 = 0.5$ and $\frac{k_{N+1}}{k_1} = 0.4$ (table VII(b)). The assumed airfoil coordinates for $N = 20$ are presented in columns (1) and (2) of the following table:

(1)	(2)	(3)	(4)
x	z/t	λ	$\Delta\lambda$
		0	
0	0		1.82120
		1.82120	
.05	.09106		-.17320
		1.64800	
.10	.17346		-.17680
		1.47120	
.15	.24702		-.18080
		1.29040	
.20	.31154		-.18400
		1.10640	
.25	.36686		-.18720
		.91920	
.30	.41282		-.19060
		.72860	
.35	.44925		-.19320
		.53540	
.40	.47602		-.19620
		.33920	
.45	.49298		-.19880
		.14040	
.50	.50000		-.20100
		-.06060	
.55	.49697		-.20360
		-.26420	
.60	.48376		-.20560
		-.46980	
.65	.46027		-.20780
		-.67760	
.70	.42639		-.20960
		-.88720	
.75	.38203		-.21140
		-1.09860	
.80	.32710		-.21320
		-1.31180	
.85	.26151		-.21480
		-1.52660	
.90	.18518		-.21640
		-1.74300	
.95	.09803		-.21760
		-1.96060	
1.00	0		1.96060
		0	
		$\sum \lambda = 0$	$\sum \Delta\lambda = 0$

The values of λ (column (3)) are differences in successive values of z/t from column (2) divided by differences in successive values of x from column (1); the values of $\Delta\lambda$ (column (4)) are differences in successive values of λ from column (3). As a check at this point, the sum of the values of λ and $\Delta\lambda$, columns (3) and (4), respectively, must individually equal zero.

The drag is, according to the scheme of equation (2),

$$
\begin{aligned}
-\frac{\beta C_D}{t^2} = {} & \Delta\lambda_1(0.2422916) + \Delta\lambda_2(0.1766747) + \Delta\lambda_3(0.5540392) + \\
& \Delta\lambda_4(0.8895163) + \Delta\lambda_5(1.1827260) + \Delta\lambda_6(1.4334612) + \\
& \Delta\lambda_7(1.6415074) + \Delta\lambda_8(1.8065982) + \Delta\lambda_9(1.9286344) + \\
& \Delta\lambda_{10}(2.0074207) + \Delta\lambda_{11}(2.0428388) + \Delta\lambda_{12}(2.0348463) + \\
& \Delta\lambda_{13}(1.9833051) + \Delta\lambda_{14}(1.8881943) + \Delta\lambda_{15}(1.7494389) + \\
& \Delta\lambda_{16}(1.5670383) + \Delta\lambda_{17}(1.3409793) + \Delta\lambda_{18}(1.0712379) + \\
& \Delta\lambda_{19}(0.7578198) + \Delta\lambda_{20}(0.4007190)
\end{aligned}
$$

or

$$\frac{\beta C_D}{t^2} = 5.7462$$

The entire computation of $\beta C_D/t^2$ starting from the tabulated coordinates should, in general, take about 1 hour.

The tabulated values (tables I to VIII) can be applied directly to airfoils having 2, 4, 5, or 10 sides by letting the appropriate values of $\Delta\lambda$ vanish. For example, if the airfoil is a double wedge with maximum thickness at the 50-percent-chord line, then $\Delta\lambda_1 = \Delta\lambda_{21} = 1$, $\Delta\lambda_{11} = -2$, and all remaining $\Delta\lambda$'s = 0.

The value of $\beta C_D/t^2$ obtained by replacing the actual airfoil section by straight-line segments depends, of course, upon the number of sides N used in the approximation. On the basis of calculations discussed in references 2 and 3, a value of $N = 20$ appears to be sufficiently large for reasonable airfoils when the linear-theory drag is finite. However, this approximate procedure would fail in all cases involving a round-nose airfoil on a wing having a supersonic leading edge inasmuch as the linear-theory drag for such cases is infinite.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 1, 1954.

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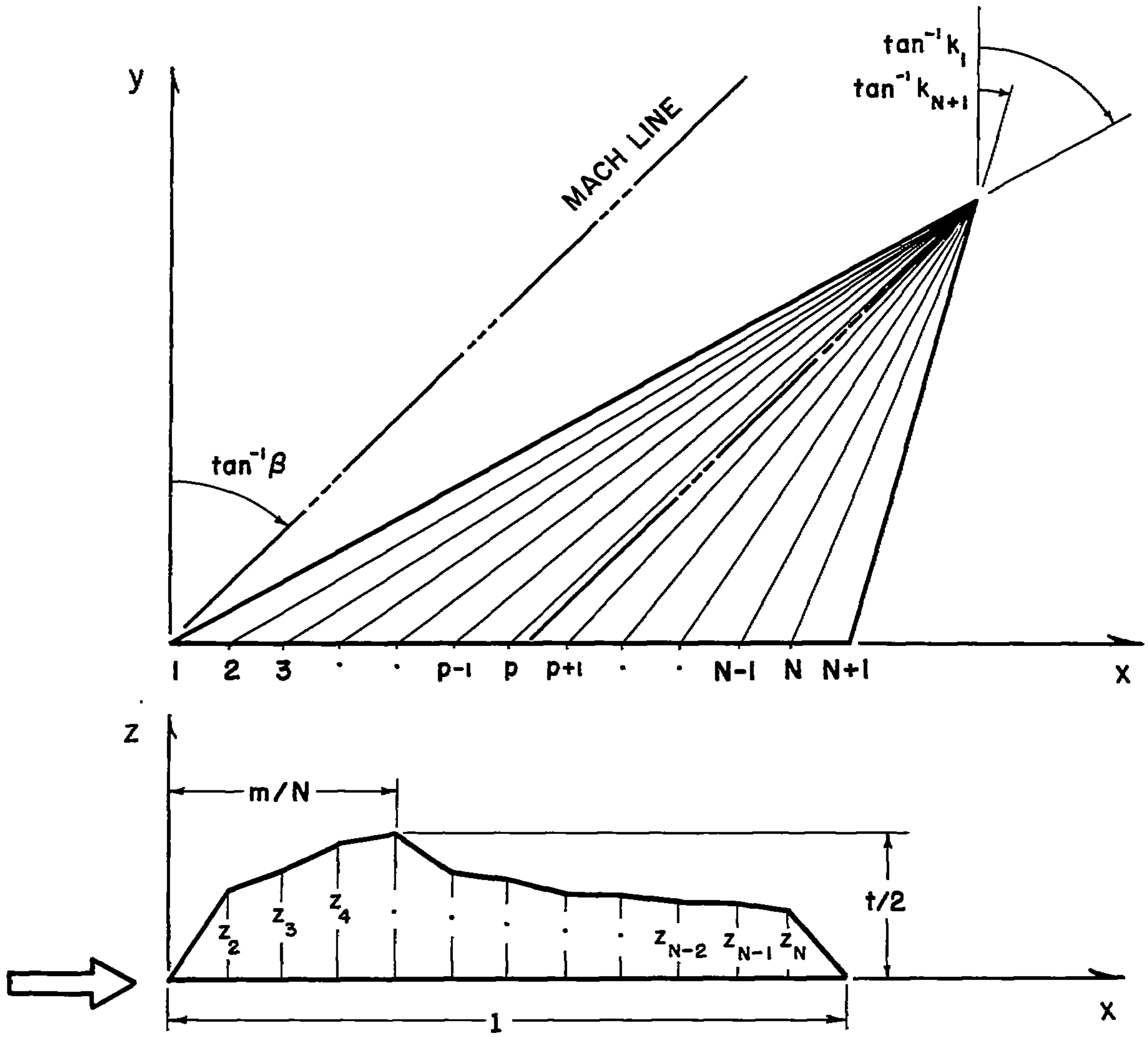

Figure 1.- Schematic drawing of wing geometry.